15.482 Healthcare Finance
Spring 2017

Andrew W. Lo, MIT

Unit 5, Part 3: Monte Carlo Simulations

Unit Outline

- Options
- Option Pricing Models
- Real Options
- Monte Carlo Simulation

Monte Carlo Simulations

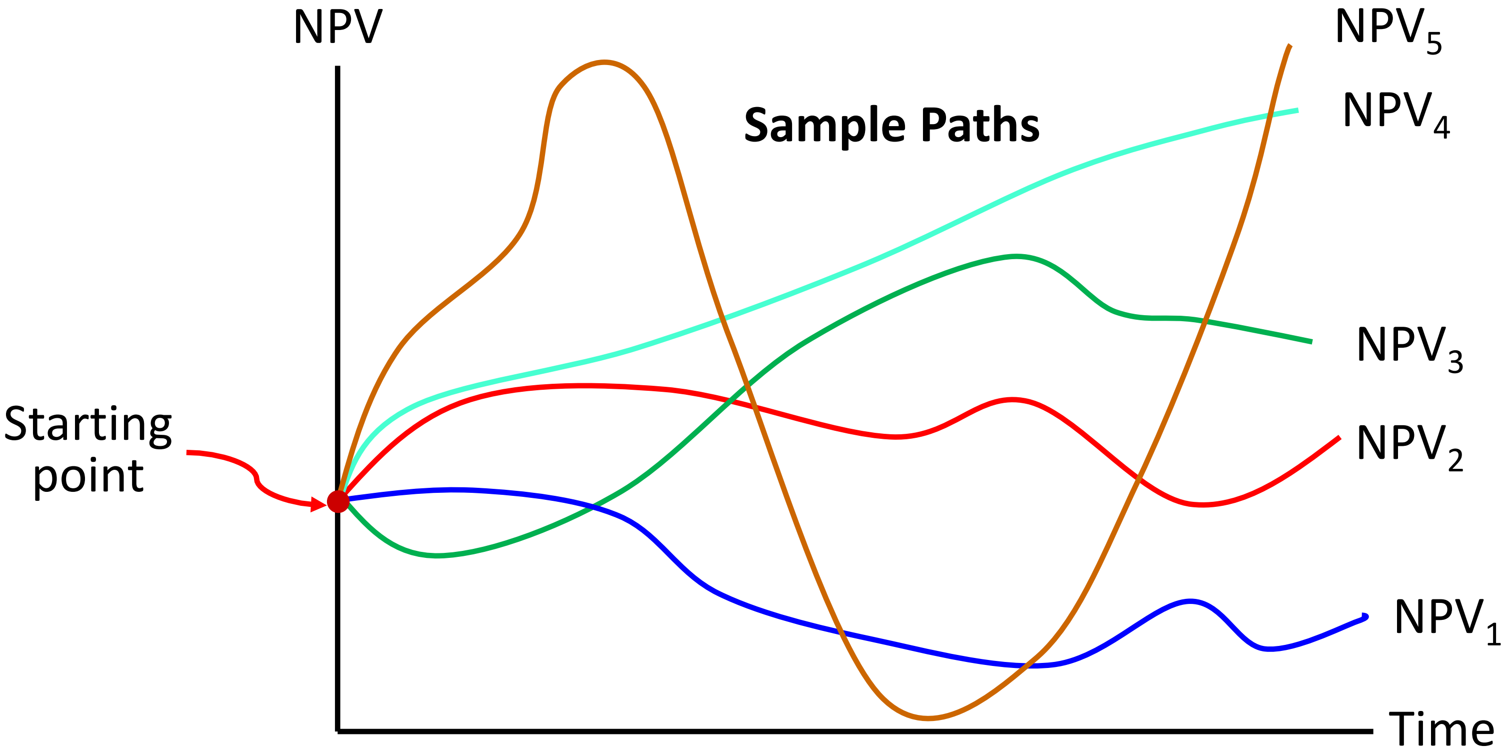
Monte Carlo Simulation

- In many cases, randomness isn't just binary, e.g., treatment effect of experimental therapies, production costs, market prices, impact of competitors, etc.
- Also, decisions may be complex nonlinear functions of several input variables, e.g., production scale, which diseases or therapeutic pathways to target and how much to devote to them, etc.
- In these cases, decision trees may be too simplistic and more sophisticated methods may be required: Monte Carlo simulation

Monte Carlo Primer

- Model the source of randomness as a particular series of random variables with specific parameters
- Propose a specific decision rule based on these random variables
- Generate one **draw** of this **sequence** of random variables, which yields a **realized sample path** of history and the outcome of the decision rule
- Do this many times to yield a **distribution** of outcomes
- Evaluate this distribution; if unattractive, change the decision rule and re-simulate to get a new distribution of outcomes

Monte Carlo Primer



Monte Carlo Primer

- Suppose we model a project as a sequence of cash flows $\{X_t\}$, but where cash flows depend on decisions $\{Y_t\}$ and random variables $\{Z_t\}$:

$$X_0(Y_0, Z_0)\ ,\ X_1(Y_1, Z_1)\ ,\ \cdots\ ,\ X_T(Y_T, Z_T)$$

$$\Downarrow$$

$$\text{rNPV} \;=\; X_0(Y_0, Z_0) + \frac{X_1(Y_1, Z_1)}{(1+R)} + \cdots + \frac{X_T(Y_T, Z_T)}{(1+R)^T}$$

- Each sample path and sequence of decisions leads to a realized rNPV; now do this many times to get multiple sample paths

Monte Carlo Primer

$$\begin{array}{lcccccc}
\text{Sample path 1:} & X_0^1, & X_1^1, & \ldots, & X_T^1 & \Rightarrow & \text{rNPV}^1 \\
\text{Sample path 2:} & X_0^2, & X_1^2, & \ldots, & X_T^2 & \Rightarrow & \text{rNPV}^2 \\
\vdots & \vdots & \vdots & & \vdots & \vdots & \vdots \\
\text{Sample path } m\text{:} & X_0^m, & X_1^m, & \ldots, & X_T^m & \Rightarrow & \text{rNPV}^m
\end{array}$$

- Given {rNPV^1, rNPV^2,…,rNPV^m} we can estimate E[rNPV], SD[rNPV], Min[rNPV], Max[rNPV], etc. (how??)

Monte Carlo Primer

$$
\begin{aligned}
\mathrm{E}[\mathrm{rNPV}] &\approx \widehat{\mathrm{E}}[\mathrm{rNPV}] = \frac{1}{m}\sum_{k=1}^{m}\mathrm{rNPV}^k \\
\mathrm{Var}[\mathrm{rNPV}] &\approx \widehat{\mathrm{Var}}[\mathrm{rNPV}] = \frac{1}{m-1}\sum_{k=1}^{m}(\mathrm{rNPV}^k - \mathrm{E}[\mathrm{rNPV}])^2 \\
\vdots \quad &\approx \quad \vdots \quad = \quad \vdots
\end{aligned}
$$

- Given the distribution of rNPV, any of its statistical properties can be estimated!
- Approximations are excellent because of LLN and CLT

Simple Real Option Example Revisited

Suppose we wish to purchase manufacturing facilities to produce a drug for which we just received FDA approval. The net profit per year X_t is \$200MM today, but next year new pricing legislation will affect this vaue, and it will remain the same thereafter. The value is a normal random variable with mean \$200MM and standard deviation of \$50MM.

- The cost of the facilities is \$1.5B today or \$1.65B next year. Should we purchase it today or wait a year? Assume a discount rate of 10% per year.
- What's the risk of losing money?
- If the decision is to wait a year, what's the chance of outperforming the NPV of an immediate purchase?
- What if the SD of profits is \$150MM?

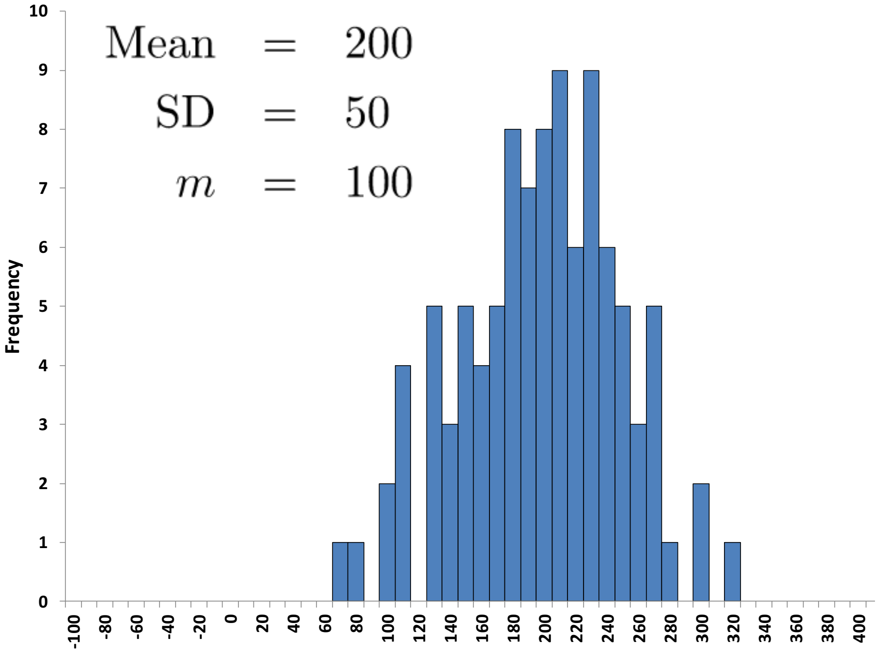
Simple Real Option Example Revisited

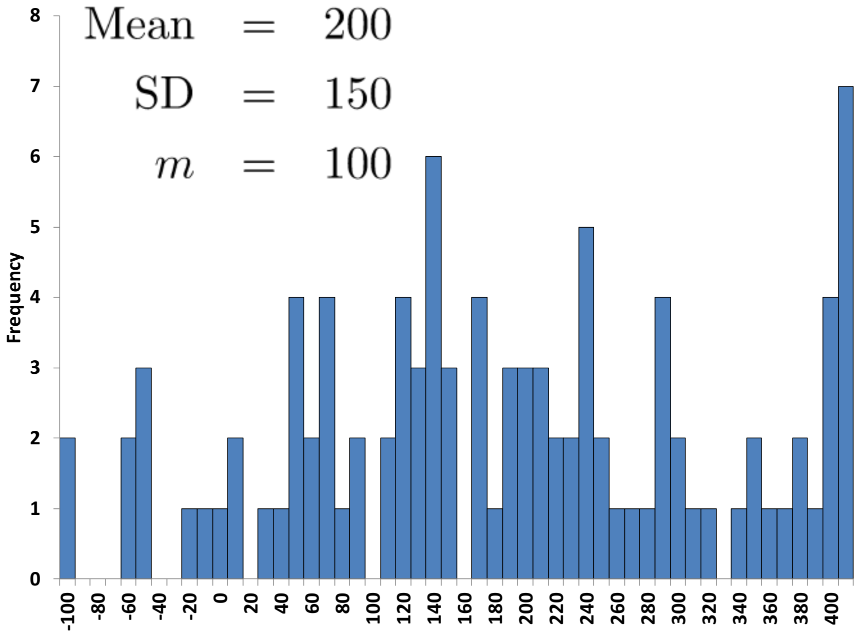
- Simulate 100 realizations of X_1
- Propose a decision rule for each realization:

$$\text{rNPV} \quad = \mathrm{E}\left(\mathrm{Max}\left[\frac{-1,650}{1.10} + \sum_{k=1}^{\infty} \frac{X_k}{1.10^k} \; , \; 0\right]\right)$$

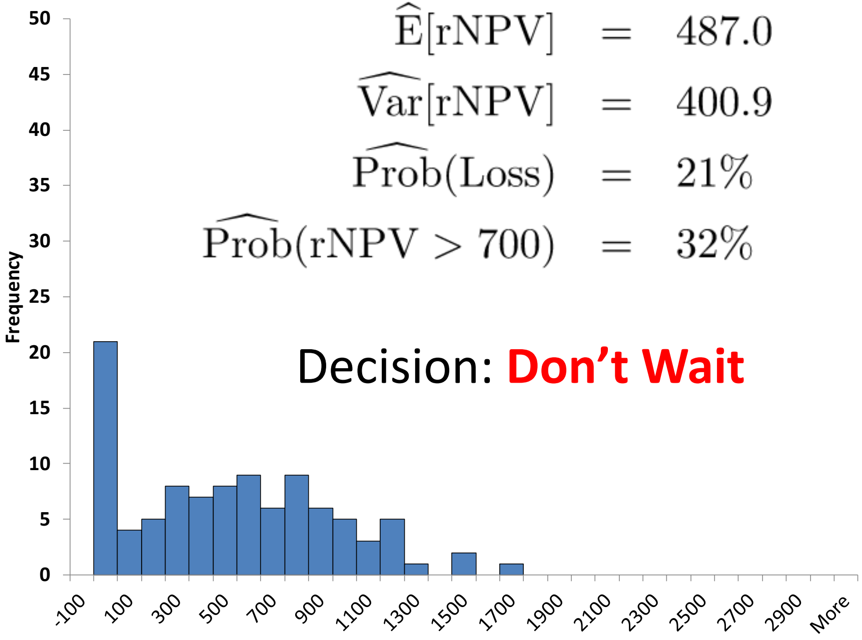
- Analyze properties of 100 rNPV’s
- Make decision based on the properties of this simulated distribution
- In this case, buy facility now or wait a year

Simulation Results of Profits



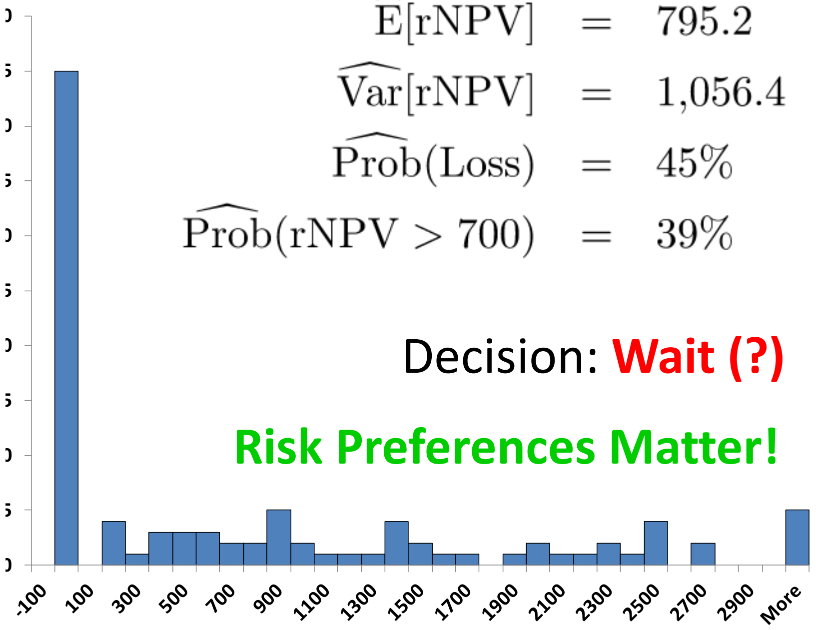


Simulation Results of rNPV



$$\begin{aligned}\hat{\mathrm{E}}[\mathrm{rNPV}] &= 487.0\\ \widehat{\mathrm{Var}}[\mathrm{rNPV}] &= 400.9\\ \widehat{\mathrm{Prob}}(\mathrm{Loss}) &= 21\%\\ \widehat{\mathrm{Prob}}(\mathrm{rNPV} > 700) &= 32\%\end{aligned}$$

Decision: **Don't Wait**



$$\begin{aligned}\hat{\mathrm{E}}[\mathrm{rNPV}] &= 795.2\\ \widehat{\mathrm{Var}}[\mathrm{rNPV}] &= 1{,}056.4\\ \widehat{\mathrm{Prob}}(\mathrm{Loss}) &= 45\%\\ \widehat{\mathrm{Prob}}(\mathrm{rNPV} > 700) &= 39\%\end{aligned}$$

Decision: **Wait (?)**

Risk Preferences Matter!

How Many Simulations Are Enough?

- Note that estimate of E[rNPV] has sampling error that can be estimated via CLT:

$$\sqrt{m}\left(\widehat{\mathrm{E}}[\mathrm{rNPV}] - \mathrm{E}[\mathrm{rNPV}]\right) \overset{a}{\sim} \mathcal{N}(0, \sigma^2_{\mathrm{rNPV}})$$

$$\mathrm{SE}\left(\widehat{\mathrm{E}}[\mathrm{rNPV}]\right) \approx \frac{\sigma_{\mathrm{rNPV}}}{\sqrt{m}}$$

- Choose *m* to the desired level of accuracy
- In this example:

$$\mathrm{SE}\left(\widehat{\mathrm{E}}[\mathrm{rNPV}]\right) \approx \frac{\sqrt{400.9}}{\sqrt{100}} = 2.00 \quad , \quad \mathrm{SE}\left(\widehat{\mathrm{E}}[\mathrm{rNPV}]\right) \approx \frac{\sqrt{1{,}056.4}}{\sqrt{100}} = 3.25$$

- Accuracy is good enough for current purposes

Summary of Monte Carlo Simulation

- Define target (object of interest, e.g., rNPV, loss, etc.)
- Specify data-generating process (DGP) and source of randomness
- Calibrate parameters of the simulations
- Simulate many sample paths of $\{X_1,\ldots,X_T\}$
  - For each sample path, implement decision rule $\{Y_1,\ldots,Y_T\}$
  - Then compute object(s) of interest
- Analyze (simulated) distribution of object of interest
- Decide whether or not decision rule yield attractive properties
- If so, done; if not, propose new decision rule and re-simulate