15.482 Healthcare Finance
Spring 2017

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Unit 5, Part 1: Option Pricing Theory

Unit Outline

- Options
- Option Pricing Models
- Real Options
- Monte Carlo Simulation

Options

Origins


The Pricing of Options and Corporate Liabilities

Fischer Black

Myron Scholes


Theory of rational option pricing

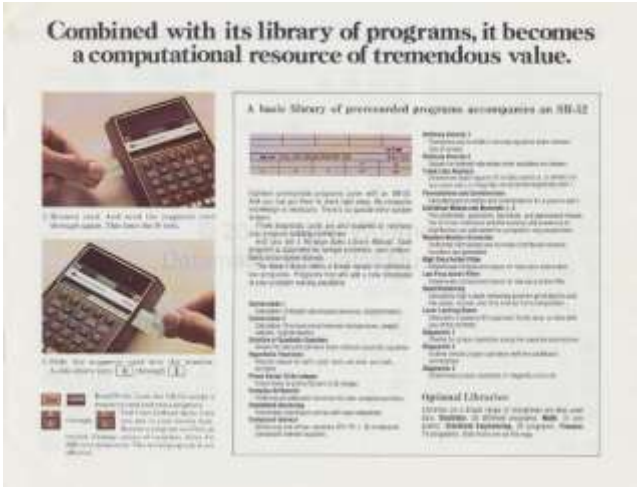
May/June 1973

CBOE
CHICAGO BOARD OPTIONS EXCHANGE

- 1973: CBOE began trading April 26, 1973
- 1975: Computerized price reporting
  Options Clearing Corporation formed
  Black-Scholes model officially adopted

Origins

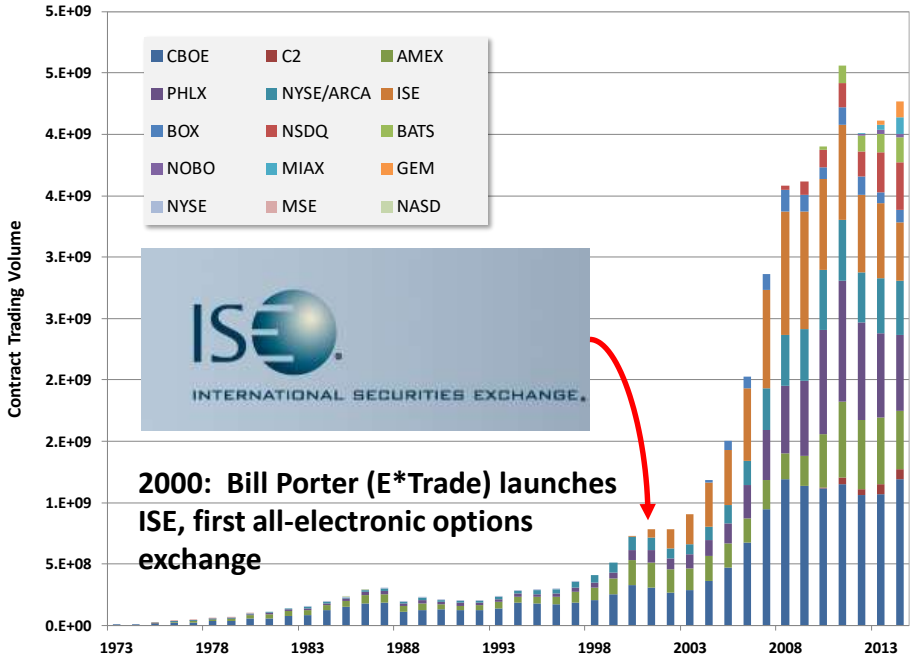
- Cost $400 in 1975
- $1,805.49 in 2017 dollars!



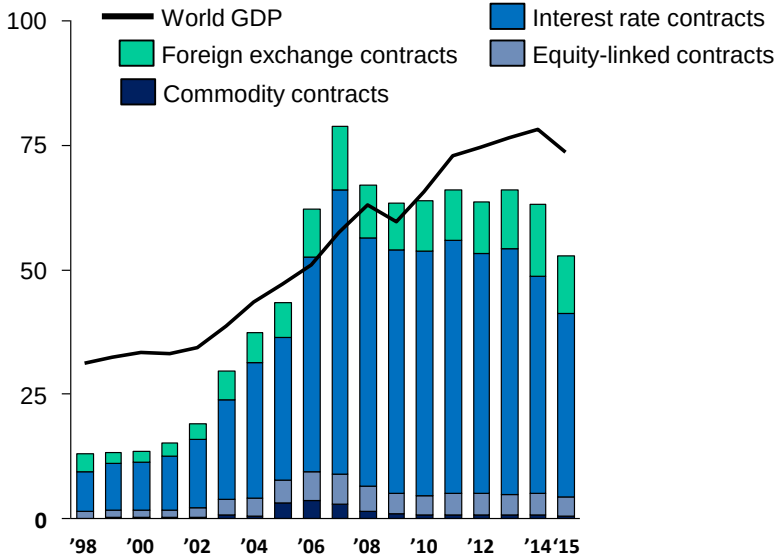
Source: Eric Rosenfeld.

Origins

Annual U.S. Options Trading Volume



OTC Derivatives Notional Amounts Outstanding in December (US$ Tr.)



Origins

1997: Nobel Prizes for Scholes and Merton

What Is An Option?

Derivative: Claim Whose Payoff is a Function of Another's

- **Warrants**
- **Call** vs. **Put Options**
- **American** vs. **European**
- Terms: **Strike Price** *K*, **Time to Maturity** $\tau = T - t$

$$S_t = \text{Stock Price} \quad , \quad K = \text{Strike Price}$$
$$C_t = \text{Call Price} \quad , \quad P_t = \text{Put Price}$$

$$C_T = \text{Max}\,[\,0\,,\; S_T - K\,]$$
$$P_T = \text{Max}\,[\,0\,,\; K - S_T\,]$$

- At Maturity *T*, payoff

What Is An Option?

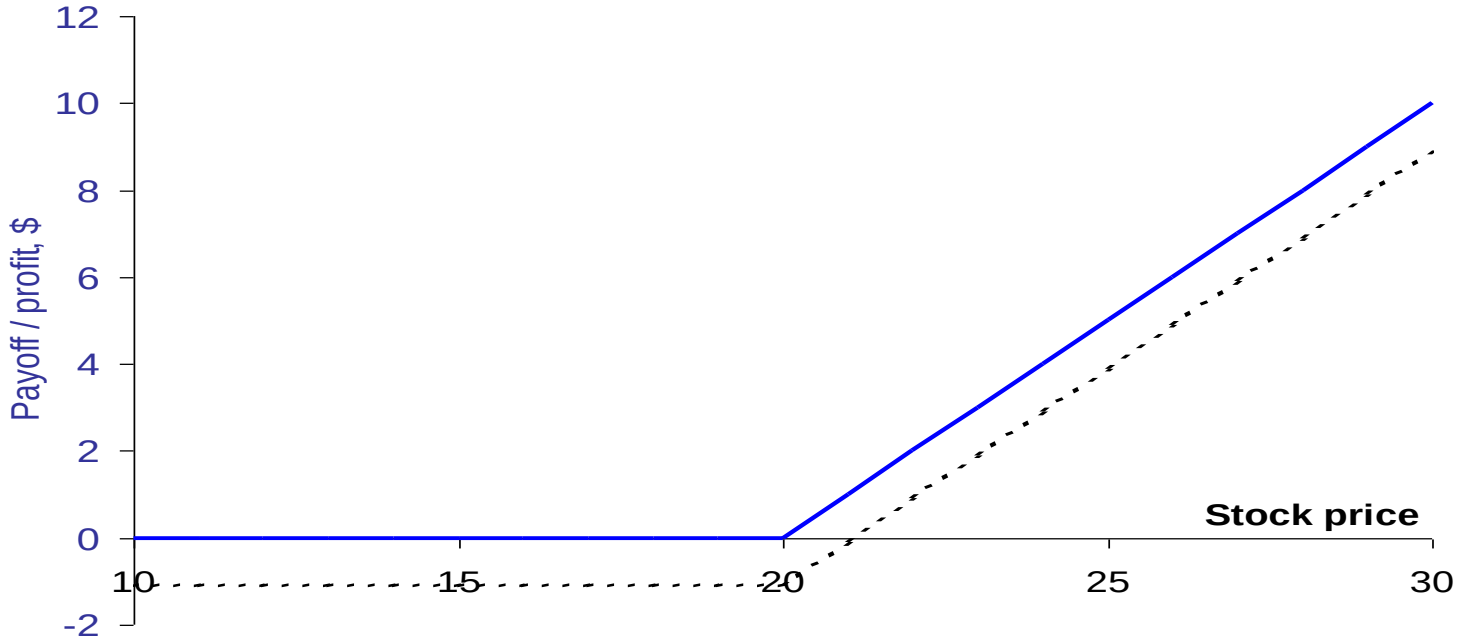
Put Options As Insurance

Asset Insured	=	Stock
Current Asset Value	=	
Term of Policy	=	
Maximum Coverage	=	
Deductible	=	
Insurance Premium	=	

- Differences
  - Early exercise
  - Marketability
  - Dividends

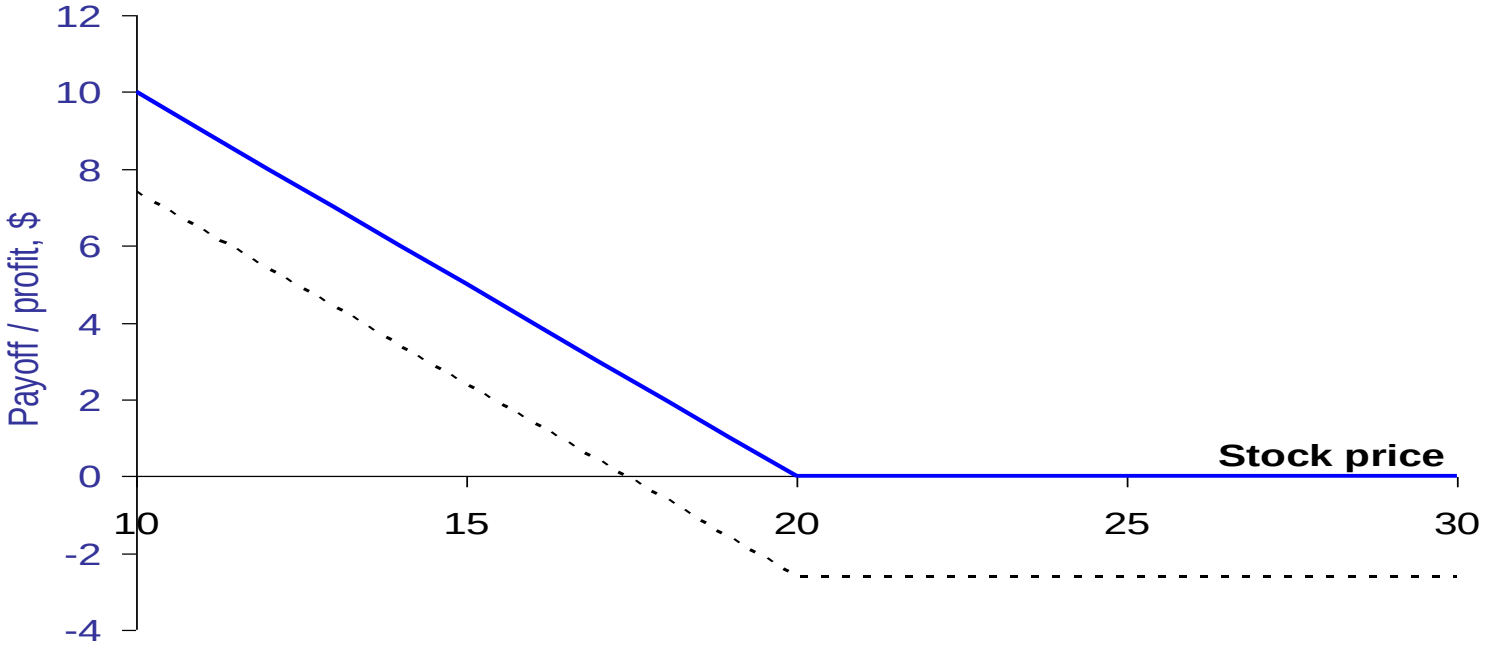
Payoff Diagrams

Call Option:



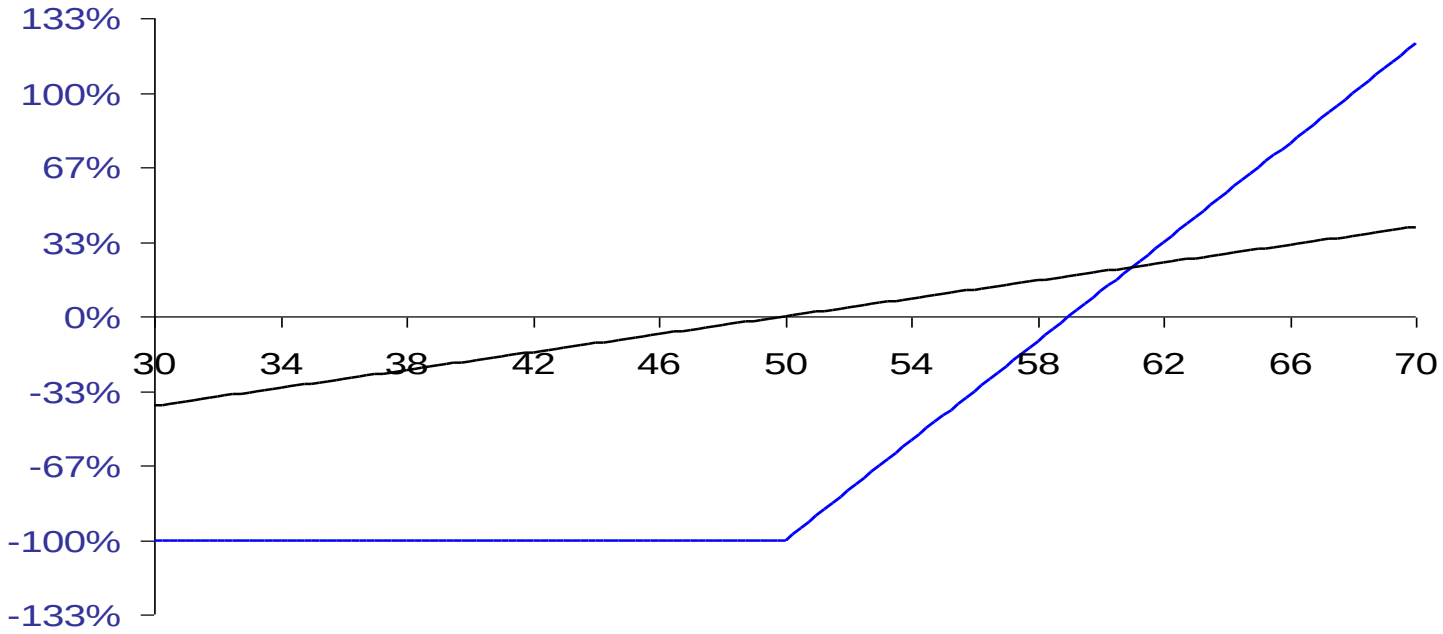
Payoff Diagrams

Put Option:

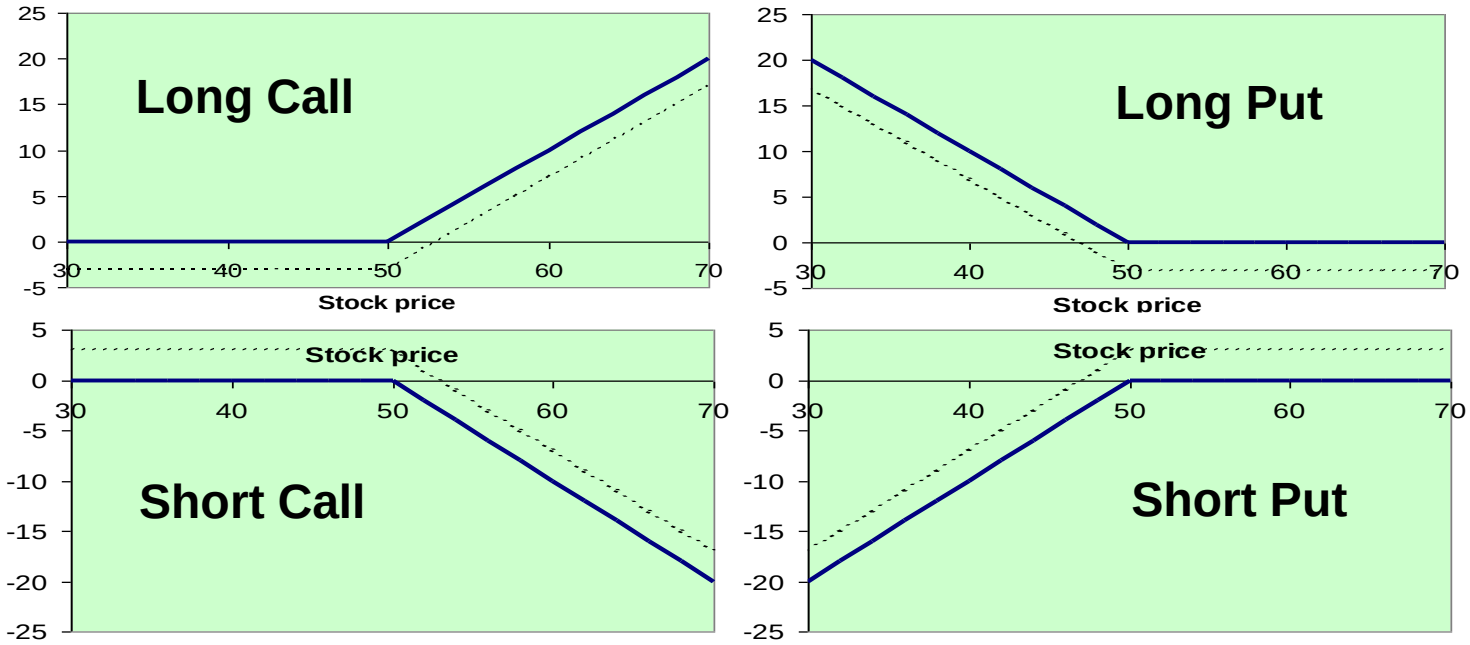


Payoff Diagrams

Stock Return vs. Call Option Return



Payoff Diagrams



Payoff Tables

Stock Price = S, Strike Price = X

Call option (price = C)

	if S < X	if S = X	if S > X
Payoff	0	0	S – X
Profit	–C	–C	S – X – C

Put option (price = P)

	if S < X	if S = X	if S > X
Payoff	X – S	0	0
Profit	X – S – P	–P	–P

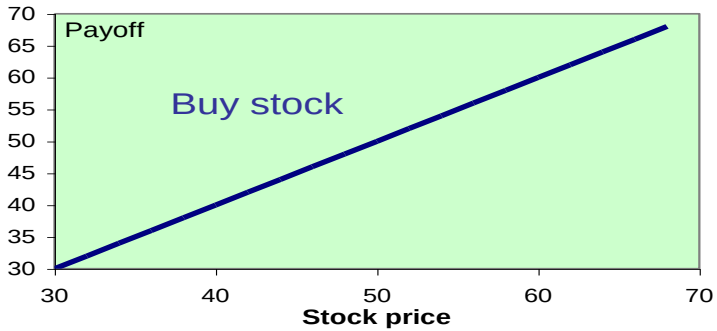
Option Strategies

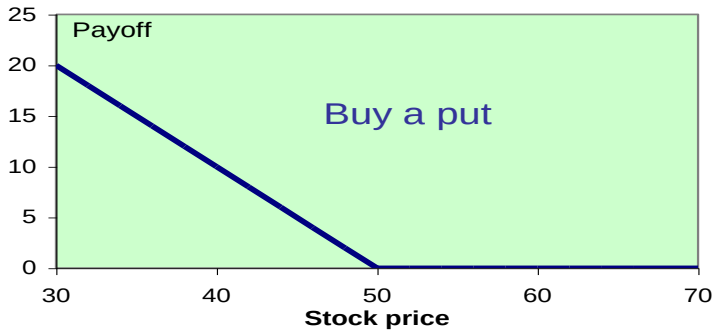
Trading Strategies

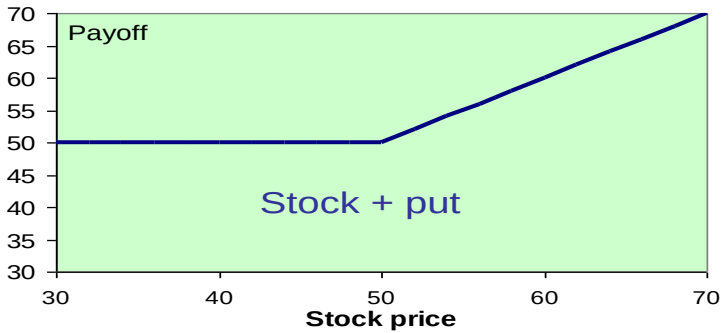
- Options can be combined in various ways to create an unlimited number of payoff profiles.
- Examples
  - Buy a stock and a put
  - Buy a call with one strike price and sell a call with another
  - Buy a call and a put with the same strike price

Option Strategies

Stock + Put

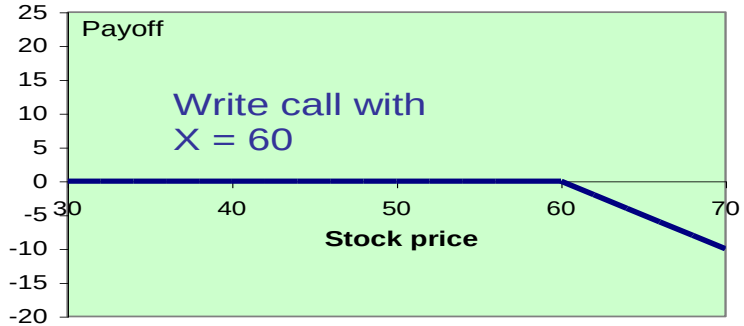


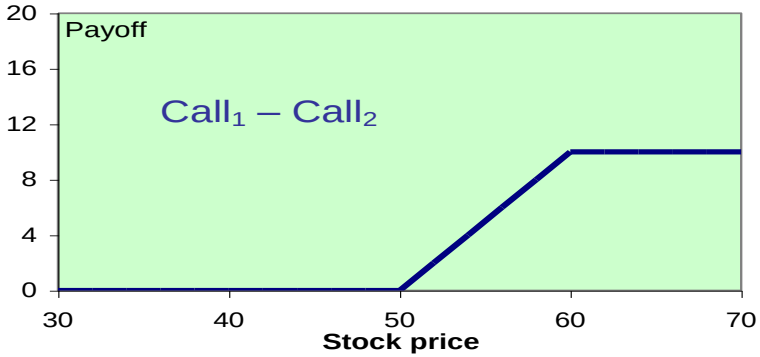




Option Strategies

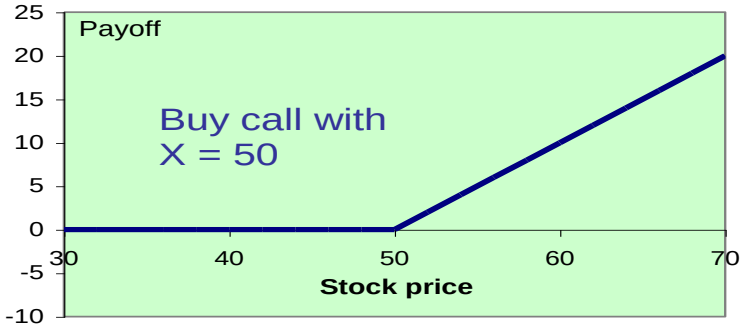
$Call_1 - Call_2$

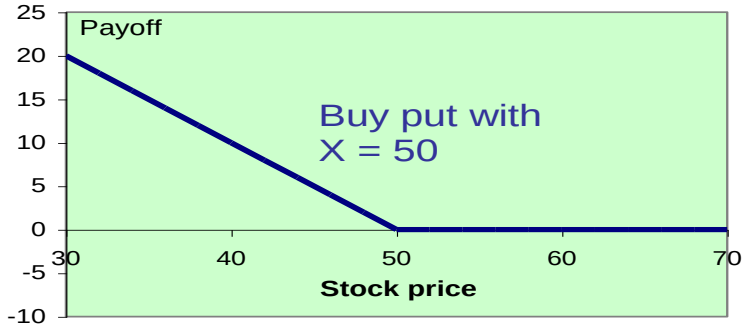


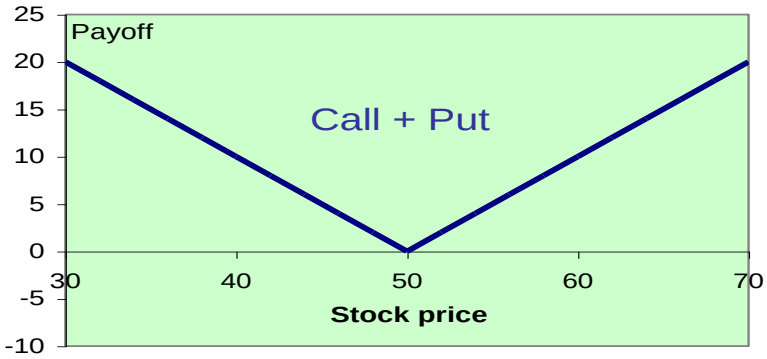


Option Strategies

Call + Put







Other Option-Like Securities

Corporate Liabilities:

$$\begin{aligned} \text{Equity} &\equiv \text{Max}\,[\,0\,,\,V-B\,] \equiv E \\ &= V - B + \text{Max}\,[\,0\,,\,B-V\,] \end{aligned}$$

$$\begin{aligned} \text{Debt} &\equiv \text{Min}\,[\,V\,,\,B\,] \equiv D \\ &= B - \text{Max}\,[\,0\,,\,B-V\,] \end{aligned}$$

$$V = D + E$$

- Equity: hold call, or protected levered put
- Debt: issue put and (riskless) lending
- Quantifies compensation incentive problems:
  - Biased towards higher risk
  - Can overwhelm NPV considerations
  - Example: leveraged equity

Other Option-Like Securities

Other Examples of Derivative Securities:

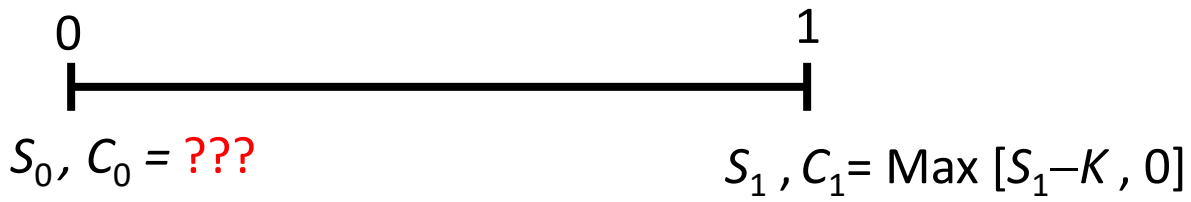
- Asian or “look back” options
- Callables, convertibles
- Futures, forwards
- Over-the-Counter (OTC) derivatives (swaps, caps, floors)
- **Patents**
- **Real investment opportunities (biomedical R&D)**
- Tenure
- etc.

Option Pricing Models

Valuation of Options

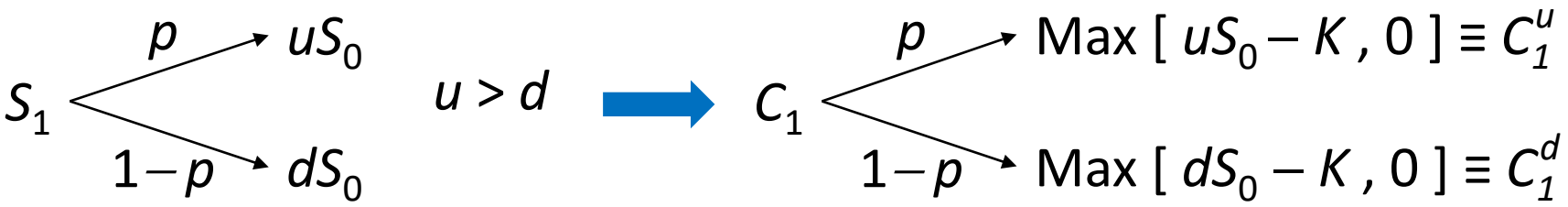
Binomial Option-Pricing Model of Cox, Ross, and Rubinstein (1979):

- Consider One-Period Call Option On Stock XYZ
- Current Stock Price S_0
- Strike Price K
- Option Expires Tomorrow, $C_1 = \text{Max}\,[S_1 - K\,,\,0]$
- What Is Today's Option Price C_0?



Valuation of Options

Suppose $S_0 \rightarrow S_1$ Is A Bernoulli Trial:



- For example, $u = 1.10$, $d = 0.90$ (S_0 moves up or down by 10%)

Now What Should $C_0 = f(\cdot)$ Depend On?

- Parameters: S_0 , K , u , d , p , r

Valuation of Options

Consider Forming a Portfolio of Stocks and Bonds Today:

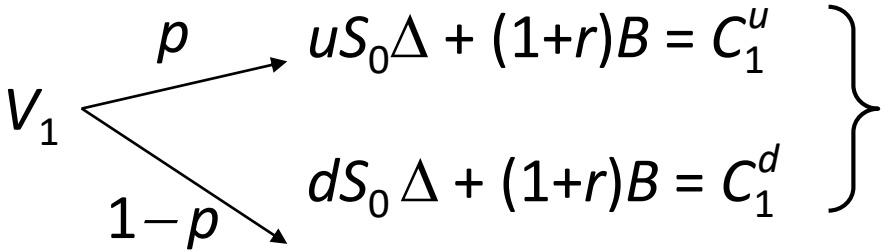
- Hold Δ shares of stock, $\$B$ of bonds
- Total cost today: $V_0 \equiv S_0\Delta + B$
- Payoff V_1 tomorrow:

$$V_1 \xrightarrow{p} V_1^u = uS_0\Delta + (1+r)B$$

$$V_1 \xrightarrow{1-p} V_1^d = dS_0\Delta + (1+r)B$$

Valuation of Options

Now Choose Δ and *B* So That:



$$\Rightarrow \quad \Delta^* = (C_1^u - C_1^d)/(u-d)S_0$$
$$B^* = (uC_1^d - dC_1^u)/(u-d)(1+r)$$

Then It Must Follow That (Why?):

$$C_0 = V_0 = S_0\Delta^* + B^* = \frac{1}{1+r}\left[\frac{(1+r)-d}{u-d}C_1^u + \frac{u-(1+r)}{u-d}C_1^d\right]$$

Valuation of Options

Suppose $C_0 > V_0$

- Today: sell C_0 , buy V_0 , receive $C_0 - V_0 > 0$
- Tomorrow: owe C_1, but V_1 is equal to C_1!

Suppose $C_0 < V_0$

- Today: Buy C_0 , sell V_0 , receive $V_0 - C_0 > 0$
- Tomorrow: owe V_1, but C_1 is equal to V_1!

$C_0 = V_0$, Otherwise Arbitrage Exists

$$C_0 = V_0 = S_0 \Delta^* + B^* = \frac{1}{1+r}\left(\frac{(1+r)-d}{u-d} C_1^u + \frac{u-(1+r)}{u-d} C_1^d \right)$$

Role of p vs. p^* and the Cost of Capital

Note That p Does Not Appear Anywhere!

- In fact, p seems completely unnecessary

$$C_0 = V_0 = S_0\,\Delta^* + B^* = \frac{1}{1+r}\left[\frac{(1+r)-d}{u-d}C_1^u + \frac{u-(1+r)}{u-d}C_1^d\right]$$

$$= \frac{1}{1+r}\left[p^* C_1^u + (1-p^*)C_1^d\right]$$

$$C_0 = \frac{1}{1+r}\mathrm{E}^*[C_1] \qquad p^* \equiv \frac{(1+r)-d}{u-d}$$

- Is p^* really a probability? Does it satisfy $0 \le p^* \le 1$

Role of *p* vs. *p** and the Cost of Capital

But *p* Does "Appear" Somewhere:

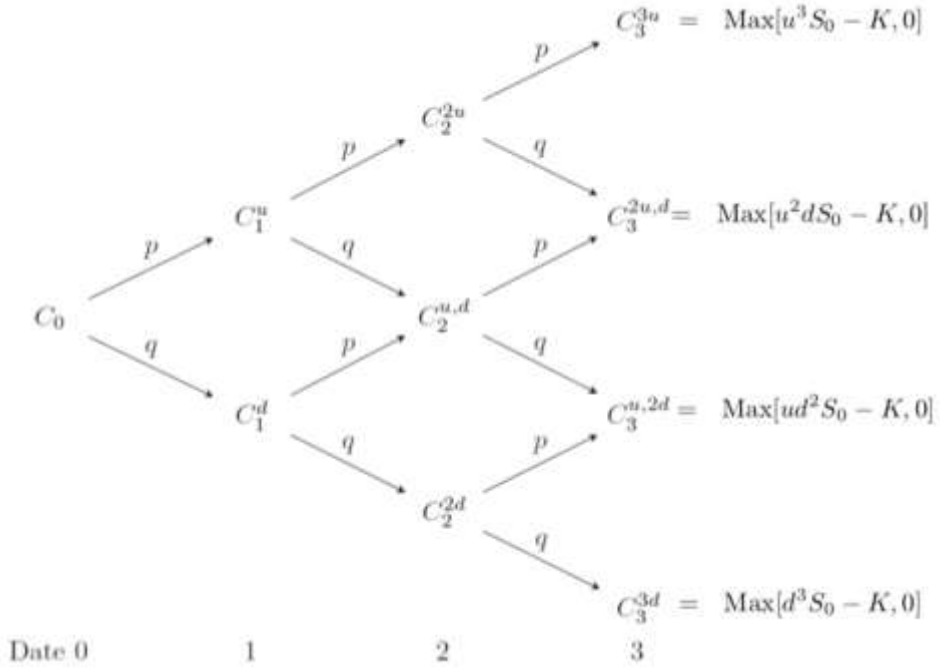
- We agree on S_0, u and d; but we know that the following relation must hold:

$$S_0 = \frac{1}{1+R} E[S_1] = \frac{1}{1+R} [\, puS_0 + (1-p)dS_0 \,]$$

- Given u and d, S_0 contains the probability p as well as the risk-adjusted cost of capital R for the asset
- Option price is completely determined by u, d, r, and S_0 but probability and risk-adjusted cost of capital is implicitly contained in these parameters
- If a 6-ounce bottle of ketchup costs $1.99, how much does a 12-ounce bottle of ketchup cost? This is a less challenging question than "how much does a 6-ounce bottle of ketchup cost?"

Multiperiod Extension

Multiperiod Version Follows Directly With Recombining Tree



$$C_0 = \frac{1}{(1+r)^n} \sum_{k=0}^{n} \binom{n}{k} p^{*k} (1-p^*)^{n-k} \times \text{Max}[0, u^k d^{n-k} S_0 - K]$$

$$p^* \equiv \frac{(1+r)-d}{u-d}$$

$$C_0 = \frac{1}{(1+r)^n} \mathsf{E}^*[C_n]$$

Multiperiod Extension

Suppose We Let $n \to \infty$, $u, d \to 1 \pm 1/n$

- Continuous-time/continuous-state version (Black-Scholes/Merton):

$$\frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} + rS\frac{\partial C}{\partial S} + \frac{\partial C}{\partial t} = rC \quad , \quad C(S,T) = \text{Max}\left[S-K, 0\right] , \; C(0,t) = 0$$

$$C(S,K,r,\tau,\sigma) = S\Phi(a_1) - K\exp(-r\tau)\Phi(a_2)$$

$$a_1 \equiv \frac{\log\left(\frac{S}{K\exp(-r\tau)}\right)}{\sigma\sqrt{\tau}} + \frac{\sigma\sqrt{\tau}}{2} \quad , \quad a_2 \equiv a_1 - \sigma\sqrt{\tau}$$

$$\Phi(\cdot) \equiv \text{Normal CDF}$$