15.482 Healthcare Finance
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Unit 1, Part 2: Time Value of Money

Unit Outline

- Market Efficiency
- The Time Value of Money
- Valuing Special Cashflows
- Inflation

The Time Value of Money

The Present Value Operator

Key Insight: cashflows at different dates are different "currencies"

- Consider foreign currencies:

$$¥150 + £300 \overset{?}{=} ??450$$

- Cannot add currencies without first converting into common currency

$$¥150 + (£300) \times (153¥/£) = ¥46{,}050.00$$

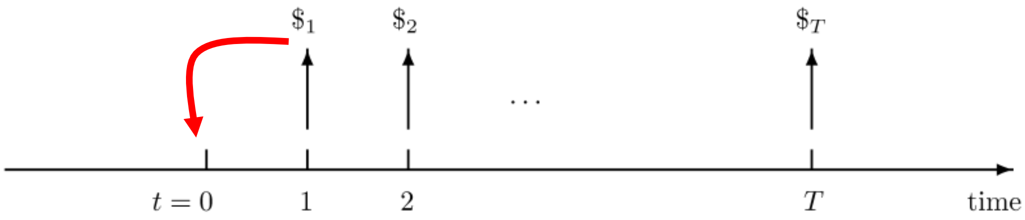
$$(¥150) \times (0.0065£/¥) + £300 = £300.98$$

- Given exchange rates, either currency can be used as "numeraire"; same idea for cashflows of different dates

The Present Value Operator

Key Insight: cashflows at different dates are different "currencies"

- Past and future cannot be combined without first converting them
- Once "exchange rates" are given, combining cashflows is trivial
- A **numeraire** date should be picked, typically t=0 or "today"
- Cashflows can then be converted to **present value**



$$V_0(\mathrm{CF}_1, \mathrm{CF}_2, \mathrm{CF}_3, \ldots) = \left(\frac{\$_0}{\$_1}\right) \times \mathrm{CF}_1 + \left(\frac{\$_0}{\$_2}\right) \times \mathrm{CF}_2 + \cdots$$

The Present Value Operator

Net Present Value: "Net" of Initial Cost or Investment

- Can be captured by date-0 cashflow CF_0

$$V_0(\mathrm{CF}_0, \mathrm{CF}_1, \ldots) \;=\; \mathrm{CF}_0 \;+\; \left(\frac{\$_0}{\$_1}\right) \times \mathrm{CF}_1 \;+\; \left(\frac{\$_0}{\$_2}\right) \times \mathrm{CF}_2 \;+\; \cdots$$

- If there is an initial investment, then $CF_0 < 0$
- Note that any CF_t can be negative (future costs)
- V_0 is a completely general expression for net present value

The Present Value Operator

Example:

- Suppose we have the following "exchange rates":

$$\left(\frac{\$_0}{\$_1}\right) = 0.90 \quad , \quad \left(\frac{\$_0}{\$_2}\right) = 0.80$$

- What is the net present value of a project requiring a current investment of \$10MM with cashflows of \$5MM in Year 1 and \$7MM in Year 2?

$$\text{NPV}_0 = -\$10 + \$5 \times 0.90 + \$7 \times 0.80 = \$0.10$$

The Time Value of Money

What Determines The Value Today of $1 In Year-T?

- $1 in year-T should be worth less than $1 today (why?)
- Supply and demand
- Consider a special case:

$$\frac{\$1}{(1+r)} \text{ in Year 0} = \$1 \text{ in Year 1}$$

$$\frac{\$1}{(1+r)^2} \text{ in Year 0} = \$1 \text{ in Year 2}$$

$$\vdots$$

$$\frac{\$1}{(1+r)^T} \text{ in Year 0} = \$1 \text{ in Year T}$$

These Are "Exchange Rates" ($\$_0/\$_t$) or Discount Factors

The Time Value of Money

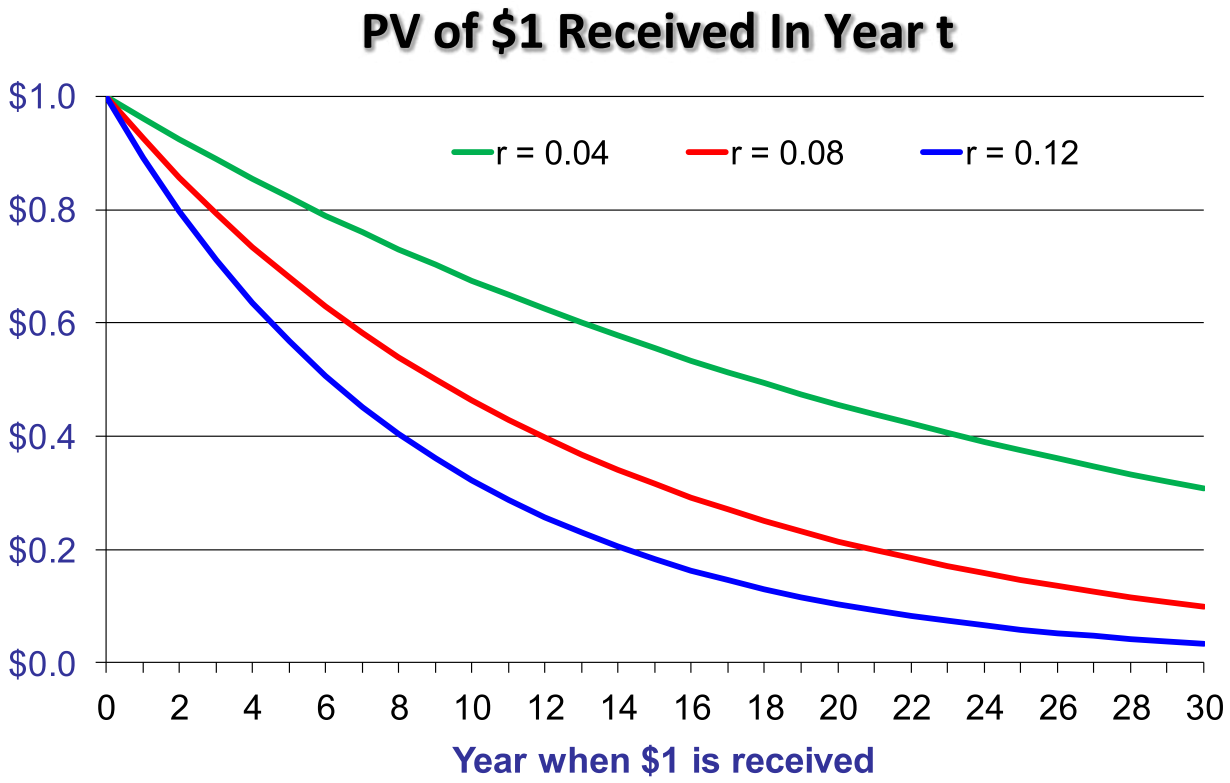
What Determines The Value Today of \$1 In Year-T?

- \$1 in year-T should be worth less than \$1 today (why?)
- Supply and demand
- Consider a special case:

$$
\begin{aligned}
\$1 \text{ in Year 0} &= \$1 \times (1+r) \quad \text{in Year 1} \\
\$1 \text{ in Year 0} &= \$1 \times (1+r)^2 \text{ in Year 2} \\
&\vdots \\
\$1 \text{ in Year 0} &= \$1 \times (1+r)^T \text{ in Year T}
\end{aligned}
$$

These Are "Exchange Rates" ($\$_0/\$_t$) or Discount Factors

The Time Value of Money



The Time Value of Money

We Now Have An Explicit Expression for V_0:

$$V_0 = \mathrm{CF}_0 + \frac{1}{(1+r)} \times \mathrm{CF}_1 + \frac{1}{(1+r)^2} \times \mathrm{CF}_2 + \cdots$$

$$V_0 = \mathrm{CF}_0 + \frac{\mathrm{CF}_1}{(1+r)} + \frac{\mathrm{CF}_2}{(1+r)^2} + \cdots$$

- Using this expression, any cashflow can be valued!
- **Take positive-NPV projects, reject negative NPV-projects**
- Projects ranked by magnitudes of NPV
- All capital budgeting and corporate finance reduces to this expression; *r* is the **cost of capital** and plays a key role

However, Many Assumptions Are Required (Perfect Markets)

Valuing Special Cashflows

Special Cashflows: The Perpetuity

Perpetuity Pays Constant Cashflow *C* Forever

- How much is an infinite cashflow of *C* each year worth?
- How can we value it?

$$\text{PV} = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \cdots$$

$$(1+r) \times \text{PV} = C + \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \cdots$$

$$r \times \text{PV} = C$$

$$\text{PV} = \frac{C}{r}$$

C = \$1,000, r = 10%

$\Rightarrow$ PV =

Special Cashflows: The Perpetuity

Growing Perpetuity Pays Growing Cashflow $C(1+g)^t$ Forever

- How much is an infinite growing cashflow of *C* each year worth?
- How can we value it?

$$\text{PV} = \frac{C}{(1+r)} + \frac{C(1+g)}{(1+r)^2} + \frac{C(1+g)^2}{(1+r)^3} + \cdots$$

$$\frac{(1+r)}{(1+g)} \times \text{PV} = \frac{C}{(1+g)} + \frac{C}{(1+r)} + \frac{C(1+g)}{(1+r)^2} + \cdots$$

$$\left[\frac{(1+r)}{(1+g)} - 1\right] \times \text{PV} = \frac{C}{(1+g)}$$

$$\text{PV} = \frac{C}{r-g} \quad , \quad r > g$$

C = \$1,000,
r = 10%, *g* = 5%
$\Rightarrow$
PV =

Special Cashflows: The Perpetuity

Growing Perpetuity Pays Growing Cashflow $C(1+g)^t$ Forever

- Growth rate is key: small changes can have huge impact
- Examples: Internet bubble; Puma Biotechnology (7/22/14)

Puma Biotechnology Announces Positive Top Line Results from Phase III PB272 Trial in Adjuvant Breast Cancer (ExteNET Trial)

Neratinib Achieves Statistically Significant Improvement in Disease Free Survival
Company Plans to File for Regulatory Approval in First Half of 2015

Example: Is Amgen Over/Under Valued?*

- Net cash flow 2016: 10.35B
- 5-year growth rate forecast: 6.77%
- Amgen cost of capital: 12.55% (http://gurufocus.com)
- Shares outstanding: 738MM

$$V = \frac{\text{NCF}}{r-g} = \frac{\$10.35 \times 10^9}{0.1255 - 0.0677} = \$179.1 \text{ billion}$$

$$P = \frac{V}{\text{Shares}} = \$242.64$$

- Is this plausible?? Current share price is $167.89

*All data are as of 2/10/17 and from finance.yahoo.com unless otherwise indicated.

Special Cashflows: The Annuity

Annuity Pays Constant Cashflow *C* For T Periods

- Simple application of V_0

$$\text{PV} = \frac{C}{(1+r)} + \cdots + \frac{C}{(1+r)^T}$$

$$(1+r) \times \text{PV} = C + \frac{C}{(1+r)} + \frac{C}{(1+r)^{T-1}}$$

$$r \times \text{PV} = C - \frac{C}{(1+r)^T}$$

$$\text{PV} = \frac{C}{r} - \frac{C}{r}\frac{1}{(1+r)^T}$$

Special Cashflows: The Annuity

Annuity Pays Constant Cashflow *C* For T Periods

- Related to perpetuity formula

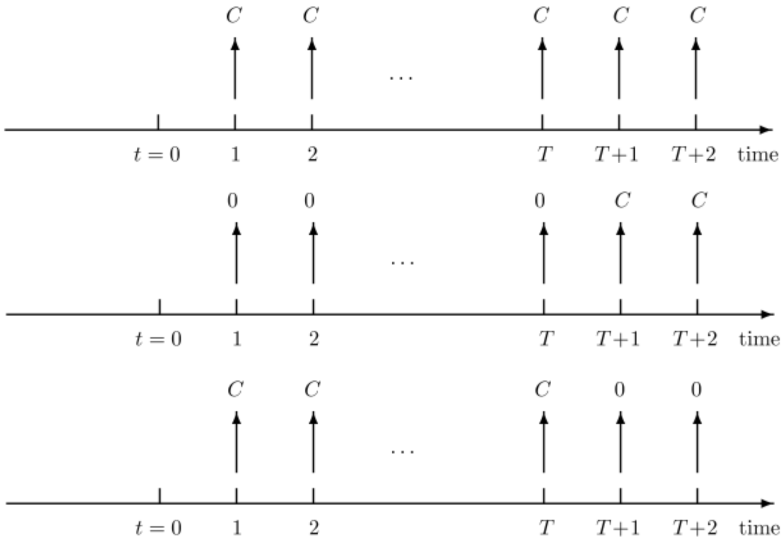
Perpetuity

Minus

Date-T Perpetuity

Equals

T-Period Annuity



Summary of Pricing Formulas

$$\text{PV} = \frac{C}{r}$$ **Perpetuity**

$$\text{PV} = \frac{C}{r} - \frac{C}{r}\frac{1}{(1+r)^T}$$ **Annuity**

$$\text{PV} = \frac{C}{r-g} \quad , \quad r > g$$ **Gordon Growth Model**

- If C is constant and known $\Rightarrow$ bonds
- If C is random $\Rightarrow$ stocks